St Nicholas School

Progression for Calculation

A Teaching Progression for Calculation

Introduction

All stages of calculation need to build upon the children's knowledge and understanding. If stages are omitted or introduced too quickly, errors and misconceptions develop. It is essential that our pupils have a strong understanding of each stage before progressing to the next; and that these stages are taught using a multisensory approach that includes a high level of teacher modelling and visual imagery.

This calculation policy is intended to provide:

- Consistency of methods taught across the school.
- Progression from practical and informal methods of calculation to formal written methods for each of the four operations (+, -, x, ÷).
- A guide for parents to understand their child's stages of learning.

To reinforce and build on written calculation methods, rapid recall of key number facts and mental strategies should be embedded. This is important as written calculations rely on mental strategies to process numbers efficiently. The policy provides a guide to mental strategies that support the various calculation methods.

Also, opportunities for pupils to use the four operations and learnt calculation methods to develop their problem solving skills should be provided where appropriate.

A Teaching Progression for Addition

ADDITION

0 – 3 years (8 -20 months and 16-26 months)

Practical Activities – Develop an interest and understanding of number

- Show an interest in number activities and counting e.g. indicates through speech or gesture a number during songs.
- Singing and games – Pupils join in with number rhymes, stories and songs. They take part in number games.
- To show an awareness of contrasting quantities when using everyday items e.g. 'one' and 'lots'. To use speech/symbols/gestures to indicate quantities. To make groups including matching to symbols.

0 – 3 years (22-36 months)

Practical Activities – Develop secure one-one correspondence and understanding of addition

- Match everyday items e.g. 1 hat for each person, 1 biscuit on each plate; when given 3 bowls, they can get 3 spoons.
- Count accurately 0-10:
  - Use number songs/games.
  - Use visual models to relate to numeral and objects e.g. 2, the Numicon shape and objects.

Sorting activities (White Rose Maths)

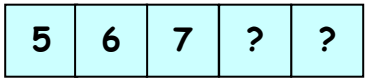
- Children to sort a range of objects such as buttons, beads, cups, cutlery, shapes. How many in each group? How are they sorted? Encourage children to also sort items from nature e.g. stones/conkers/leaves/flowers etc
- Sort numbers/object/ pictures/ visual representations.

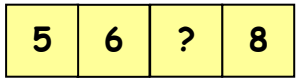
Useful books:

3- 4 years (30-50 months)

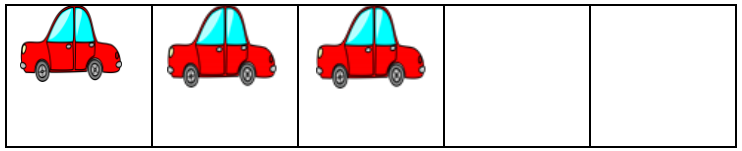
Practical Activities – Develop secure one-one correspondence and understanding of addition

- Recognise numerals 1-10 and order them:
- Number cards, Numicon, missing numbers.





- To understand the concept of more/fewer/less/bigger/smaller by comparing quantities.
- To 'add one' e.g. fetch one more chair; add one pencil to the pot.
- Encourage children to use a 5 frame. Ask the children to put a number of objects on the 5 frame. Ask them to add 1 more. How many do you have now? You can have a number track underneath the five frame. Point to the number you made. Point to the number 1 more.



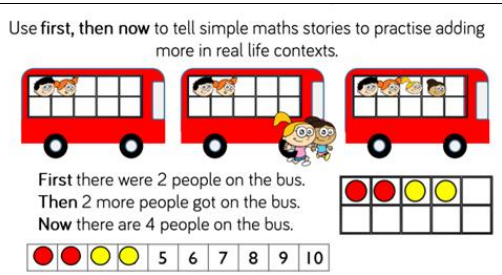
- In your outdoor area, set up some bus stops. Ask 1 child to stand at each bus stop. Adult/child it so be the driver and go round to each bus stop and count on 1 more. Encourage chn to say how many on the bus altogether.
- Play a shopping game with the children. Give each child a shopping bag. Have a collection of objects on the table. Go around the table and say '_____ went shopping and he/she brought……' Ensure the child only puts 1 item into their bag each time. How many items for they have now?
- In your construction area, build some towers of bricks. Ask the children to add on 1 more to each tower of bricks.

NB: These activities can also be used to show 1 less. For 1 less you can also sing 5 currant buns or play musical chairs where 1 chair is taken away each time. See

Frog and Toad – A Lost Button
The Button Box my M Reid
	subtraction section
	
	• To begin to count sets of real objects to 5:
	  ○ Respond to 'how many?'
	  ○ Use Numicon Shapes to support understanding of numerals and number.
	
	Books
	The Gingerbread Man
	The Enormous Turnip
	The very hungry caterpillar
	Maisy goes camping (you can also use this book to show 1 less and composition to 5. Encourage chn to use soft toys and a play tent to act out the story
	
	Songs
	5 little speckled frogs
	5 currant buns
	5 little ducks
	5 tiddly widdly tadpoles
	5 little monkeys jumping on the bed

ADDITION
Reception (40 – 60 months)

Practical and Pictorial Activities – Develop understanding of addition
• To count sets of real objects to 10:
  ○ Use Numicon as visual model.
  ○ Use numeral cards.
 	
• To find 'one before' and 'one after' numbers to 10 e.g. 'What is the number after 8? Before 6? Before 10? What is the number between 5 and 7? What number is between 3 and 5?
 	
• To add together real objects and pictures:
  ○ Consolidate 'one more' and 'add one'.
• To use addition when responding to verbal instruction relating to real objects e.g. 'get two more pencils' or 'add three more mushrooms'.
• Encourage chn to use their fingers e.g. hold up 5 fingers, hold up 2 more. How many do you have? Some children may need to count all finger; some may be able to count on from last number spoken e.g. 5, 6, 7.
• Use a number track to support adding in real life contexts using First, Then, Now e.g. First 2 children were on a bus, Then 2 more got on the bus, Now there are 4 on the bus.



Vocabulary

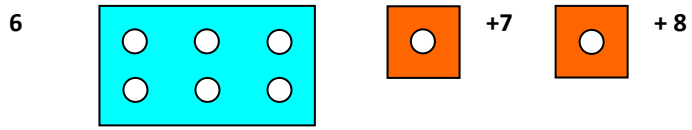
- Before
- After
- More
- Add
- Group

- Consolidate adding objects to 10 e.g. 'add two more', 'four more' e.g. play a number track race . Each child is to have a number track 1-10 and a dice 1-3. Children take turns to roll the dice and then count on 1, 2 or 3. The first child to 10 is the winner. This can also be done outside or in PE with children jumping along giant tracks
- Construction area: The children take turns to throw a 1-3 dice and then take that number of bricks/blocks to build a tower. Children can count how many or count on each time. How high can they build a tower before it topples over?
- Small world: Encourage the children to create their own First, Then, Now stories e.g. First there were 2 dolls in the house, then 3 more came for tea. Now there are 5 dolls in the house.
- Relate to pictorial representation: 4 + 2 = 6  +
- Encourage pupils to record pictorially using written marks or stamps.

Books

Mouse Count
Mr Grumpy's Outing
The Shopping Basket

- To use fingers and objects (Numicon Shapes) to count on: 'You have 6 faces, add 2 more'.



6 in head, count on 2: 6, 7, 8

- To know that addition is the combining of 2 groups of objects (*use Numicon and pictures to support).*
- To begin to use vocabulary of addition in practical situations e.g. 'add more'.

ADDITION

Bridging Levels 1 and 2

+ and = signs – Develop understanding of addition and the number line

- To recognise addition and relate it to the combining of two numbers (*use groups of objects, counting on, Numicon Shapes and pictorial representation).*
- Children also need to understand that numbers can be partitioned into 2 or more parts. This will help with number bonds and addition.
- Children will need to be introduced to the addition symbol (+) and combine it with the equal symbols (=). Chn need to focus on the specific order of the sentence (a+b=c) and on the language to the sentence e.g. 7 apples plus 3 apples equals 10 apples. First, then, now stories (see earlier) will help this

Vocabulary

- Add
- More
- Count on
- Plus
- Sum
- Total
- Altogether
- How many
- Number line
- 100 square
- Part
- Whole

- To know addition can be done in any order:
  - **3 + 2** is the same as **2 + 3.**
  - Use objects, pictures and Numicon Shapes as visual models.
- To begin to count on. Chn to use objects/cubes/numicon/pictorial representations. Encourage children to only count on from the bigger number e.g. 3 + 5 using cubes. Encourage children to count out only 3 cubes (smaller numbers and count on from 5 rather than counting out all the cubes).
- Children can also build on number sentences by looking at addition fact families. They can see that the order of the number sentence can be varied and can discover that addition is commutative e.g. 3+2=5, 2+3 = 5, 5=2+3, 5=3+2

Number Bonds within 10

- Children combine their knowledge of the part-whole model and addition facts to explore number bonds within 10. Starting with the whole, children break numbers into parts and explore how many different ways numbers can be partitioned e.g. 5 = 3+2, 5=4+1
- Children apply their partitioning skills to work systematically starting with the whole e.g. 7+0=7, 6+1=7, 5+2=7, 3+4=7. This is supported through the use of equipment, for example cubes, bead, strings, double sided counters.

Bridging 3 and Year 1

Number Bonds to 10 and 20

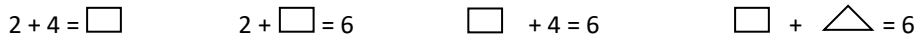
- Focusing on the number 10, children use a variety of representations to explore number bonds to 10 systematically e.g. ten frames, fingers, numicon. The children should also see the number sentence alongside the representation to help further develop their conceptual understanding.
- Children will see that working systematically will help them find number bonds to 20.They will use their knowledge of number bonds to 10 to find number bonds to 20.

Add ones

- Children should see a pattern when we add and comment on what happens. The pattern should also be highlighted by adding 2 and 3 etc.

Missing Numbers

- To be able to complete number sentences where missing numbers are shown by a symbol or gap:



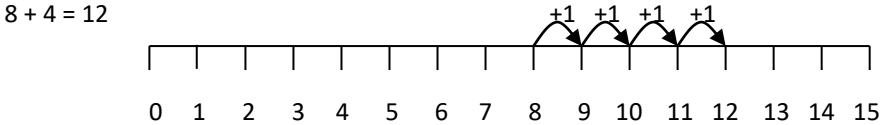
- Children should apply their understanding of number bonds to solve missing number problems. Building on from counting on, children should start from the given part and count on to the whole, to find the missing part.

Number Lines

- To use a prepared number line to add one or more numbers to 10:

  - Encourage pupils to record jumps if able (pupils could stick on prepared jumps made from paper, string, elastic bands or pipe cleaners).
  - Teachers to model – use Numicon number lines as additional support if needed.

- To use a prepared number line to add one or more numbers beyond 10:



  - Encourage pupils to record jumps if able (pupils could stick on prepared jumps made from paper, string, elastic bands or pipe cleaners).
  - Teachers to model – use Numicon number lines as additional support if needed.

- To be able to use a hundred square to add two numbers together including TU + U:

23 + 6 =

- To be able to add multiples of 10 to any number to 100 using a hundred square:

34 + 10 = 44 Increase this to **34 + 50 = 84**

ADDITION

Year 2

Fact Families

- Children apply their understanding of known addition and subtraction facts within 20 to identify all related facts. This will include an understanding of the relationship between addition and subtraction, and knowing the purpose the equals sign, as well as addition and subtraction signs.

Check Calculations

- Children need to have opportunities to discuss and share strategies to for checking addition and subtraction calculations. This does not mean just the inverse but also using concrete resources, number lines and estimating.

Related Facts

- Children should have an understanding of calculations with similar digits e.g. 2+5=7, 20+50=70. 'Tens' and 'ones' can also be used to aid understanding. Base 10 can help children see the relationship.

Number Bonds to 100 (tens)

- Focus on multiples of 10 up to 100. Links should be made between the single digit and 10s digit e.g. 3+7=10, 30+70=100

10 more and adding 10

- Children need to focus on the importance of the tens digit. Use a 100 square and explore what happens to the number in the columns e.g. the tens number changes but the unit number stays the same. Use concrete materials before going onto abstract ideas.
- Children should make use of place value to add tens from a given number within 100.

+ and = signs

- To continue solving a range of number sentences with missing numbers as in Phase 4, but with appropriate larger numbers.
  E.g. 8 + ☐ = 13 13 + 6 = 10 + ☐ ☐ + ☐ = 13

  and adding 3 numbers: 5 + ☐ + 3 = 15 or 26 = 2 + ☐ +6

- To continue to add multiples of 10 using a hundred square, number line and Numicon Shapes e.g. 34 + 20 =

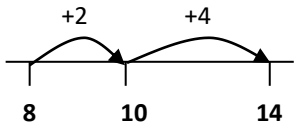
- To understand that addition is the inverse of subtraction: **5 + 3 = 8** → **8 - 5 = 3**

Vocabulary

- Add
- More
- Count on
- Plus
- Sum
- Total
- Altogether
- Partition
- How many
- Multiple of 10
- Number line
- 100 square

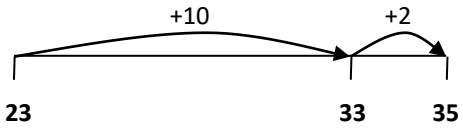
Add 2 digit + 1 digit; adding 2 digit number and tens; adding 2 digit numbers

- To be able to use partitioning in order to bridge through 10: **8 + 6 = 14**



  - Begin to introduce empty number line.
  - Include using a hundred square.

- To partition the second number only e.g. **23 + 12 = 23 + 10 + 2**
  = 33 + 2
  = 35



Visual models to support each step:

- Prepared number line
- Empty number line
- Hundred square
- Numicon Shapes
- Diennes

- To partition into tens and ones and recombine (*use diennes and Numicon for as visual models)*:

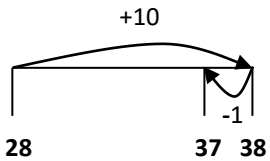
12 + 24 = 10 + 2 + 20 + 4
= 10 + 20
= 2 + 4
= 30 + 6
= 36

ADDITION

Year 2 continued

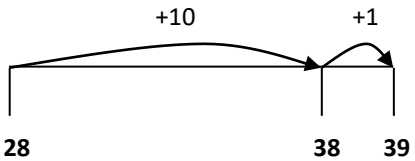
- To add 9 or 11 by adding 10 and adjusting by 1.

E.g. **28 + 9 = 37**



or

28 + 11 = 39



Number Bonds to 100 (tens and ones)

- Children need to build on their earlier work on number bonds to 100 with tens together with number bonds to 10 and 20. They use their knowledge of exchange to find number bonds to 100 with tens and ones. Using 100 squares, bead strings, numicon etc will support children.

Add three 1 digit numbers

- Children need to use their idea of commutativity to find the most efficient and quick way to add the three one-digit numbers. Some may be able to use their knowledge of number bonds to do it quickly.

ADDITION

Year 3

Add and Subtract multiples of 100

- Children are introduced to adding numbers greater than 100 by using concrete manipulatives and pictorial representations throughout. Children need to see the value of the digits.

3 digits and 1-digit numbers (with and without an exchange) and 3 digit and 2-digit numbers (with and without an exchange)

- Children add ones from a 3-digit number without exchange. They consider which digits are affected when adding ones. They should be encouraged to use both the column method and mental strategies.
- Children move onto adding ones to a 3-digit number, with an exchange. They discover that when adding ones it can affect the ones column and the tens column. Children learn that we can only hold single digits in each column, anything over must be exchanged. They also need to be aware of the use of 0 as a place holder.
- Children also look at hat happens to a 3-digit number when a multiple of 10 is added without an exchange. Use base 10, arrow care, place value charts etc to represent this. Ideally, children should use mental strategies for this but they can also use the column method.
- Children move on to adding multiples of 10 to a 3-digit number with an exchange. They recognise that when adding tens, it can change the tens and hundreds column. Children should be encouraged to use mental strategies first (counting in tens) before looking at the column method.

Add 100s

- Children build on their knowledge of adding 100s together e.g. 30+500, by adding ones and tens to solve calculations such as 234+500.

+ and = signs

- To continue solving a range of number sentences with missing numbers as in Phase 4 and 5, but with appropriate numbers.

  E.g. $18 + \square = 35$ $\quad 20 + \square + 40 = 100$ $\quad 246 + \square = 346$

  $\square + 17 = 35$ $\quad \square + \square + \square = 100$

- To partition into hundreds, tens and ones and in different ways:
  - To partition the second number only

    36 + 53 = 53 + 30 + 6

Visual models to support earlier steps:

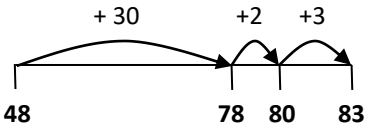
- Prepared number line
- Empty number line
- Hundred square
- Numicon Shapes
- Diennes

= 83 + 6
= 89

Record on a number line.

- To split one-digit numbers to bridge a multiple of 10

48 + 35 = 83



- To continue to add a near multiple of 10 to a 2-digit number.

Develop skills acquired in Phase 5 but with appropriate numbers:

E.g. **34 + 19** is the same as **34 + 20 - 1**

ADDITION

Year 3

Pencil and Paper Procedures

- To record partitioned numbers under one another:

27 + 14 =	**20**	**7**	
	10	**4**	
	30	**11**	**= 30 + 11 = 41**

OR

346 + 123 =	**300**	**40**	**6**	
	100	**20**	**3**	
	400	**60**	**9**	**= 469**

Vocabulary

- **As above**
- Column
- Addition
- Tens
- Boundary

Adding 2-digit and 3-digit numbers

- Children focus on the position of numbers and place value to add and subtract 2 digit and 3-digit numbers. They represent using Base 10 and line up the place value columns. Children should be adding without exchange.
- Children deepen their understanding of adding 2-digit and 3-digit numbers in this step. They start adding numbers where there is an exchange from ones to tens, they then move on to exchanging tens and hundreds before adding number where there are exchanges in both columns. Sue concrete representations and the column method to support children in

understanding how the column method works.

Adding two 3-digit numbers

- Children add two 3-digit numbers with no exchange. They should focus on the lining up of the digits and setting the additions clearly in columns.
- Children add two 3-digit number with an exchange. They start by adding numbers where there is one exchange required before looking at questions where they need to exchange in two different columns. Children may use Base 10 or place value counters to model their understanding. Ensure children continue to show the written method alongside the concrete so they understand when and why an exchange takes place.

Estimating and Checking Answers

- Children check how reasonable their answers are. Discuss why estimations are important. Use real life examples where children and adults need to estimate.
- Children explore ways of checking to see if an answer is reasonable. Checking using inverse is to be encouraged so that the children are using a different method and not just potentially repeating an error, for example, if they add in a different order.

ADDITION
Year 4

Adding 1s, 10s, 100s, 1000s

- Children build on prior learning of adding hundreds, tens and ones by being introduced to adding thousands. They can continue to use concrete representations (base 10, place value counters etc) before moving to abstract and mental methods.

Add two 4-digit numbers

- Children use their understanding of addition of 3-digit numbers to add two 4-dgit numbers with no exchange. They can use concrete equipment and a place value grid to support their understanding alongside column addition.
- Children then move onto the adding two 4-digit numbers with one exchange. They use a place value grid to support understanding alongside column addition.
- Children will then begin to explore multiple exchanges within an addition of 4-digit numbers. Children should continue to use equipment alongside the written method to help secure understanding of why exchanges take place and how we record them.

Estimate Answers and Checking Strategies

- Children use their knowledge of rounding to estimate answers for calculations and word problems.
- Children explore ways of checking to see if an answer is correct by using inverse operations.

+ and = signs

- To continue solving a range of number sentences with missing numbers as in Phases 4 - 6, but with appropriate numbers.
  E.g. 53 + ☐ = 100
  20 + 60 + ☐ = 120

484 + ☐ = 684

- To add the nearest multiple of 10 and then adjust.

  Develop skills acquired in Phase 5/6 but with appropriate numbers:

  E.g. **64 + 29 = 64 + 30 – 1**

Pencil and Paper Procedures – Expanded Column Method

- To add two-digit numbers in vertical layout:
  - Add the tens first

    37
    + 86
    110
    13
    123

  - Add the ones first

    37
    + 86
    13
    110
    123

  - Discuss how adding ones first gives the same answer as adding the tens first. Lead to adding the ones first consistently.

Vocabulary
- **As above**
- Hundreds
- Boundary
- Approximate

ADDITION
Year 4

- To add 3-digit numbers in vertical layout:

367 + 184 = 551

367
+184
11
140
400
551

Vocabulary
- **As above**
- Carry
- Decimal

- To use the formal column method (*use with more able and only when previous phases and steps are secure):*

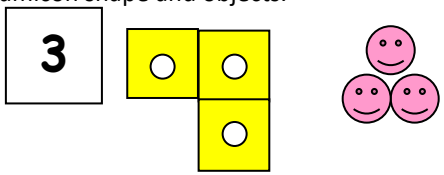
```
  T U
  2 8        'Carrying' below the line.
+ 1 7
  4 5
  |
```

Extension

- To add decimals in the context of money (vertically):

£3.50 + £1.75 = £5.25

```
  £3.50
+ £1.75      Ensure decimal point remains in line.
  £5.25
  |
```

SUBTRACTION		
0 – 3 years (16–26 months)	**0 – 3 years** (22-36 months)	**3- 4 years** (30-50 months)
Practical Activities – Develop an interest and understanding of number • Show an interest in number activities and counting e.g. indicates through speech or gesture a number during songs. • Singing and games – Pupils join in with number rhymes, stories and songs. They take part in number games.	**Practical Activities – Develop secure one-one correspondence and understanding of subtraction** • Count accurately forwards and backwards from 0-5 and then 0-10: o Use number songs/games. o Use visual models to relate to numeral and objects e.g. 3, the Numicon shape and objects.  • Recognise, record and order numerals 1-5 and then 1-10: o Number cards, Numicon, missing numbers.  • Singing rhymes such as Five Little Speckled Frog, Five Currant Buns. • Books: Five Little Ducks by Denise Fleming, Five Tiddly Widdly Tadpoles by Debbie Tarbett and Five Little Monkeys Jumping on the bed by Eileen Christelow	**Practical Activities – Develop secure one-one correspondence and understanding of subtraction** • Children use their counting and comparing skills to find one less than numbers up to 5. Children can use a five frame to represent numbers and then make one less. Children should see the link that one less than a number is the next number they say when they are counting backwards. • To understand the concept of more/fewer/less/bigger/smaller by comparing quantities. • To find 'one less' than a number from 1 – 10 (*Use numbers songs such as 'Ten in a bed' and 'Ten Green Bottles'*). • To 'take one' e.g. take one plate away; take one pencil out of the pot. • To use subtraction when responding to verbal instruction relating to real objects e.g. 'take away 3 cups' or 'give me 4 less mushrooms'. **Vocabulary** ❖ Less ❖ Take away ❖ More ❖ Fewer ❖ Bigger ❖ Smaller

Reception (40 – 60 months)

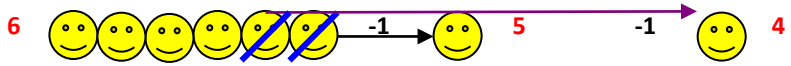
Practical and Pictorial Activities – Develop understanding of addition

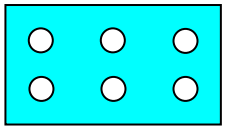
- To find 'one before' and 'one after' numbers to 10 e.g. 'What is the number after 8? Before 6? Before 10? What is the number between 5 and 7? What number is between 3 and 5? What number is between 7 and 10?

- To count and subtract real objects and pictures:
  - Consolidate 'one less' and 'take one'.
  - Consolidate taking away objects to 10 e.g. 'take away two', 'four less'.
  - Relate to pictorial representation: 5 - 2 = 3 
  - Encourage pupils to record pictorially using written marks or stamps.

Vocabulary
- Before
- After
- Less
- Take away
- Count

- To use fingers and objects (Numicon Shapes) to count back: 'you have 6 faces, take away 2'.





-1 = 5 **-1 = 4**

Cover Numicon Shape with hand or overlays.

6 in head, count back 2: 6, 5, 4

- To know that subtraction is taking away objects (*use Numicon and pictures to support).*

- To begin to use vocabulary of subtraction in practical situations e.g. 'take away' or 'less'.

- Children use real objects to see that the quantity of a group can be changed by taking items away. The first, then, now structure can begin to be sued to create mathematical stories in a meaningful context. Encourage the children to count out all of the items at the start, take away the required amount practically, and recount to see how many left. Children should represent their number stories using 10 frames, number tracks and their fingers.

Other resources:
Mouse Count by Ellen Stoll Walsh
Kippers Toybox by Mick Inkpen
Incey Wincey Spider game Nrich

Bridging 1 and 2

Finding a part

- Children should apply their understanding of number bonds to solve missing number problems. Building on from counting on. Children should tart from a given part and count on to find the whole, to find the missing part. Children should also be exposed to problems with one part and the whole being the same so they understand the role of zero.

How many left?

- Children are introduced to the language of subtraction. The term 'taking away' is used in a range of real-life contexts. The use if zero is important so children know that when nothing is taken away the whole remains the same. First, then now story representations can help the children understand the concept of 'how many left'.
- Once children understand the concept of taking away, the subtraction symbols can be introduced. It is still important for children to create stories about the calculation and use concrete and pictorial representations so they can deepen their understanding of subtraction.

Breaking Apart

- Children continue to sue the subtraction symbols. Building on their understanding of finding a part, they are introduced to subtraction by partioning. Children break apart a number into two parts using concrete and pictorial representations to support.

+ and = signs – Develop understanding of subtraction and the number line

- To recognise subtraction as taking something away (*use groups of objects, counting on, Numicon Shapes and pictorial representation).*

Vocabulary

- Subtract
- Count on
- Count back
- Difference

Missing Numbers

- To be able to complete number sentences where missing numbers are shown by a symbol or gap:

7 - 3 = ☐ 7 - ☐ = 4 ☐ - 3 = 4 ☐ - △ = 4

Count Back

- Children count backwards to subtract. It is an important step to help children work in the abstract. Common misconceptions could be that children include

Their starting number when counting e.g. 5- 3, 5, 4, 3 – there giving the wrong answer. It is vital to model how to count backwards by 'putting the start number in our head and counting backwards'. Children need to be confident with this before moving on to using number lines.

Number Lines

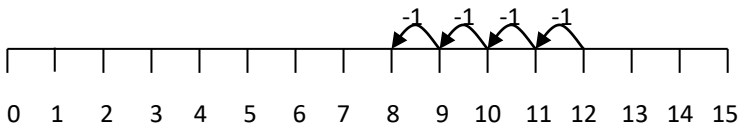
- To use a prepared number line to take one or find one less when using numbers to 10:

9 - 2 = 7

  - Pupils to take away by counting back.
  - Encourage pupils to record jumps if able (pupils could stick on prepared jumps made from paper, string, elastic bands or pipe cleaners).
  - Teachers to model – use Numicon number lines as additional support if needed.

- To use a prepared number line to take away one or find one less when using numbers beyond 10:

12 - 4 = 8



  - Pupils to take away by counting back.
  - Encourage pupils to record jumps if able (pupils could stick on prepared jumps made from paper, string, elastic bands or pipe cleaners).
  - Teachers to model – use Numicon number lines as additional support if needed.

SUBTRACTION

Bridging 3 and Year 1

Finding a difference

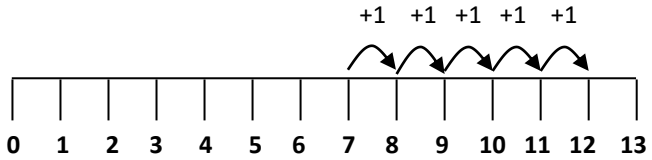
- Children explore finding the difference as a form of subtraction. They often struggle with this concept because both parts are given.

Children could use their skills of counting back and counting on to help them find the difference. Alternatively, they can make both amounts and visually see how many more/less a number is.

- To find a 'difference' by counting up:

  Find the difference between 7 and 12

  12 – 7 (counting up)



Visual models to support each step:

- Prepared number line
- Hundred square
- Numicon number line

  - Encourage pupils to record jumps if able (pupils could stick on prepared jumps made from paper, string, elastic bands or pipe cleaners).
  - More able begin to construct own lines.
  - Teachers to model – use Numicon number lines as additional support if needed.

Subtracting multiples of 10s

- To be able to subtract multiples of 10 from any number to 100 using a hundred square or prepared number line:

34 - 10 = 24

Increase this to **64 - 50 = 14**

Subtraction – not crossing 10

- Children build on the language of subtraction, recognising and using the subtraction symbol within 20. The use of zero is important so children know that when nothing is taken away, the start number remains the same or when the whole group is taken away, there will be nothing left. They will also use the part-whole model alongside practical equipment to reinforce number bonds within 20.

Subtraction – crossing 10

- Children will be introduced to subtraction where they have to cross ten. This small step focuses on the strategy if partitioning to make 10. Children should represent this using concrete manipulatives or pictorially to begin with. Ten frames and number lines are particularly useful to model the structure of this strategy. Children will move towards using this as a mental strategy.
- Children subtract numbers, within 20, crossing the 10. Children being to understand the different structures of subtraction (taking away, partitioning, difference). They use concrete manipulatives and pictorial methods to support their understanding. One of the most difficult concepts for children is finding the difference where they subtract to calculate how many more.

SUBTRACTION

Year 2

Subtract 1s

- As in adding on 1, children should also start seeing the patterns when we subtract 1 and comment upon what happens. The pattern could also be highlighted also by subtracting 2 (by subtracting another one) and then subtracting three.

10 less and subtract 10s

- As in adding 10, teaching needs to focus on the importance of the tens digit. Using a 100 square, explore with the children what happens to the numbers in the columns. Draw attention to the idea that the tens digit changes while the ones digit remains the same. Children will need to see how the number changes with concrete materials before moving onto more abstract ideas.
- Children should also make use of place value to subtract 1-s from a given number within 100. Children should be encouraged to see the relationship and understand the importance of the tens digit within given numbers.

+ and = signs

- To continue solving a range of number sentences with missing numbers as in Phase 4, but with appropriate larger numbers.
  E.g. 18 - ☐ = 14 9 + 6 = 20 - ☐

- To continue to subtract multiples of 10 using a hundred square, number line and Numicon Shapes e.g. 34 - 20 =

- To understand that addition is the inverse of subtraction:

5 + 3 = 8 ⟶ 8 - 5 = 3

- To partition into (hundreds), tens and ones and recombine (*use diennes and Numicon for as visual models)*:

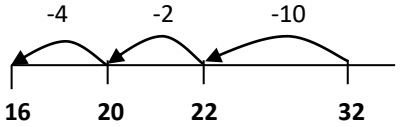
22 + 11 = 20 + 2 + 10 + 1
= 20 - 10
= 2 - 1
= 10 + 1
= 11

Subtract 1-digit from 2-digits

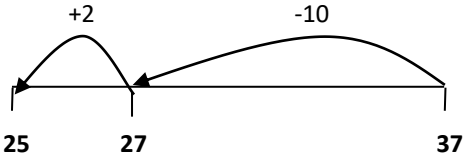
- Children need to have a strong understanding of place value. They need to be able to count to 20 and need to be able to partition two-digit numbers in order to subtract from them. They need to understand the difference between one-digit and two-digit numbers and line them up in columns. In order to progress to using the number line more efficiently, children need to be secure in their number bonds.

Number lines

- To be able to use partitioning in order to bridge through 10: **32 – 16 = 16**
  - Begin to introduce empty number line.
  - Include using a hundred square.



- To partition the second number only e.g. **37 - 12 =**
  = 37 – 10 - 2
  = 27 - 2
  = 25



Vocabulary

- Take away
- Subtract
- Count on
- Count back
- Less
- Minus
- Subtraction
- difference
- Partition
- How many
- Multiple of 10
- Number line
- 100 square

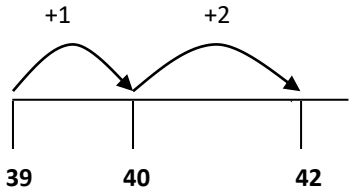
Subtract with 2-digits (not crossing 10 and then crossing 10)

- It is important to work on not crossing tens with children before looking at subtraction where they cross a tens boundary. Children need to use concrete materials but also draw images of the Base 10 so they can independently solve problems.
- Children use their knowledge that one ten is the same as ten ones to exchange when crossing a ten in subtraction. Continue to use concrete manipulatives (such as Base 10) and pictorial representations (such as number lines and part whole models) to develop the children's understanding.

Year 2

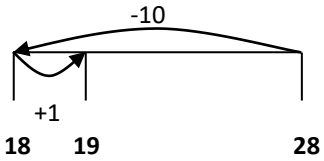
- To find a small difference by counting up:

42 – 39 =



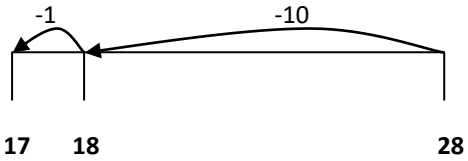
- To subtract 9 or 11 by taking away 10 and adjusting by 1.

E.g. **28 - 9 = 19**



or

28 - 11 = 17



Visual models to support each step:

- Prepared number line
- Empty number line
- Hundred square
- Numicon Shapes
- Diennes

Year 3

+ and = signs

- To continue solving a range of number sentences with missing numbers as in Phase 4 and 5, but with appropriate numbers.

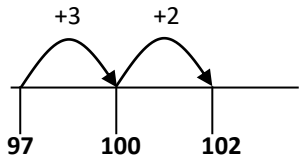
  E.g. 36 – 17 = ☐ ☐ - 15 = 19 20 - ☐ - ☐ = 5 ☐ - ☐ = 18

Visual models to support earlier steps:

- Prepared number line
- Empty number line
- Hundred square
- Numicon Shapes
- Diennes

- To find a small difference by counting up (*continue to develop skills from Phase 5*):

102 – 97 = 5



  - These number line methods for 'finding a difference' should also be used for work with money, time, measures, decimals etc.

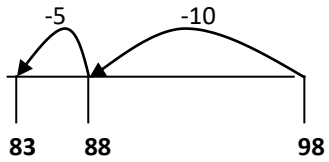
- To continue to subtract a near multiple of 10 from a 2-digit number.

  Develop skills acquired in Phase 5 but with appropriate numbers:

  E.g. **78 - 49** is the same as **78 - 50 + 1**

- To use known number facts and place value to subtract mentally:

98 – 15 = 83



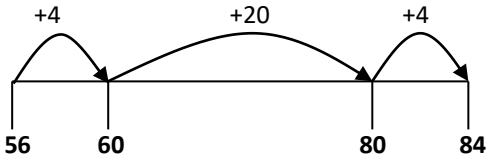
  - Ask the children whether counting up or back is the more efficient method for calculations such as **59 – 12** or **32 – 29**

Year 3

Pencil and Paper Procedures – Complimentary Addition

- To solve subtraction by using complimentary addition:

84 – 56 = 28



Vocabulary

- **As above**
- Addition
- Subtraction
- Tens
- Ones

Subtract multiples of 100

- Children are introduced to subtracting numbers greater than 100. They will apply their knowledge of subtracting ones and tens to subtracting multiples of 100. Using concrete manipulatives and pictorial representations throughout is important so the children can see the value of the digits.

3-digit and 1-digit numbers

- Children subtract ones form a 3-digit number without an exchange. They consider which digits are affected when subtracting ones e.g. is a child is completing 214-3, they will see that they just need to focus on the ones column. Therefore, all the need to do is 4-3 respectively. They can use the column method but mental arithmetic is the best strategy.

Subtract 1-digit from 3-digits.

- Children subtract 1-digit from a 3-digit number using an exchange. Children need to b secure in the fact that 321 is 3 hundred, 2 tens and 1 one but that it is also 3 hundred, 1 ten and 11 ones. If children are not secure with regrouping, it is important to revisit this before subtracting.
- Children the look at what happens to a 3 digit numbers when a multiple of 10 is subtracted. Different representations such as Base 10, arrow cards, place value charts should be used. The use of the column method is exemplified in this example, but children should d explore whether or not this is needed and explain why. Mental methods should be encouraged throughout.

Subtract 2-digits from 3-digits

- Children subtract multiples of 10 from 3-digit number, with an exchange. Counting backwards in tens or using 100 to help you will support mental strategies.

Subtract 100s

- Children build on their knowledge of subtracting 100s together e.g. 500-200, by adding ones and tens to solve calculations such as 554-200. It is important to develop flexibility and ask the children why the column method isn't always the most effective method. Highlight that when adding and subtracting 100s, the ones and tens columns are net effected.

Subtract a 2-digit and 3-digit number (not crossing 10 and 100)

- Children focus on the position of numbers and place value to subtract 2-digit and 3-digit numbers. They represent numbers using Base 10 and line up the place value columns. In this step, children subtract numbers without an exchange.

Subtract 2-digits from 3 digits with exchange

- Children focus on the position of numbers and place value to subtract 2-digits from 3-digits using the column method. Children start by exchanging one ten for ten ones.
- Next they exchange one hundred for ten tens before subtracting numbers where there are exchanges in both columns.
- Children can use Base 10 and place value counters so they can physically exchange and see the link between the concrete and the written column method.

Subtract 3-digitis from 3-digits

- It is important for the children to understand that there are different methods of subtraction. They need to explore efficient strategies for subtraction including:
  - Counting on (number lines)
  - Near subtraction
  - Number bonds
- They then move on to setting out formal column subtraction supported by practical equipment
- Children then explore column subtraction using concrete manipulatives. It is important to show the column method alongside so that children make the connection to the abstract method and so understand what is happening.
- Children progress from an exchange in one column, to an exchange in two columns. Reinforce the importance of recording any exchanges clearly in the written method.

SUBTRACTION

Year 4

+ and = signs

- To continue solving a range of number sentences with missing numbers as in Year 2 and 3, but with appropriate numbers.
  E.g. 100 - 18 = ☐
  ☐ - 15 = 70
  100 - ☐ - ☐ = 20

- To subtract the nearest multiple of 10 and then adjust.
  Develop skills acquired in Phase 5/6 but with appropriate numbers:

  E.g. **67 - 29 = 64 - 30 + 1** **OR** **67 – 31 = 67 – 30 – 1**

Vocabulary

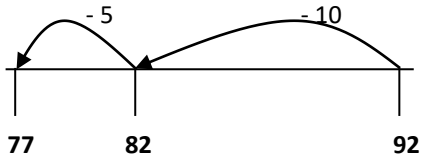
- **As above**
- Multiple
- Hundred
- Partition

Subtract s, 10s, 100s, 1000s

- Children build on prior learning of subtracting hundreds, tens and ones. They are introduced to subtracting thousands. Children should use concrete representations (Base 10, place value counters etc) before moving to abstract and mental methods.

- To use known number facts and place value to subtract:

92 – 15 = 77



OR

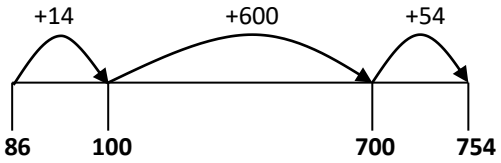
244 – 65 = ?
244 – 65 = 184
184 – 5 = 179

SUBTRACTION
Year 4

Pencil and Paper Procedures – Complimentary Addition

- To solve subtraction by using complimentary addition

754 – 86 = 668



  - Extend more able by reducing the number of steps to make the calculation more efficient.
  - The complementary addition used for 'finding the difference' should be developed as a mental strategy especially when numbers are close to boundaries e.g. 3001 – 2999.

Extension for more able

- To use the expanded partitioning layout:

77 – 24 =

	70	7	
-	20	4	
	50	3	= 50 + 3 = 53

Visual models to support earlier steps:

- Prepared number line
- Empty number line
- Hundred square
- Numicon Shapes
- Diennes

Subtract two 4-digit numbers

- Building on their experiences in year 3, children use their knowledge of subtracting using the formal column method to subtract two 4-digit numbers. Children will focus on calculations with no exchanges, concentrating on the value of each digit.
- Children then explore subtractions where there is one exchange. They use place value counters to model the exchange and match this with the written column method.
- Children then explore what happens when a subtraction has more than one exchange. They can continue to use manipulatives to support their understanding. Some children may feel confident calculating with a written method.
- Encourage children to continue to explain their working to ensure they have a secure understanding of exchange within 4-digit numbers.

Efficient Subtraction

- Children use their understanding of column subtraction and mental methods to find the most efficient methods of subtraction. They compare the different methods of subtraction and discuss whether they would partition, take away or find the difference

MULTIPLICATION

0 – 3 years (16–26 months, 22-36 months) **and 3- 4 years** (30-50 months)

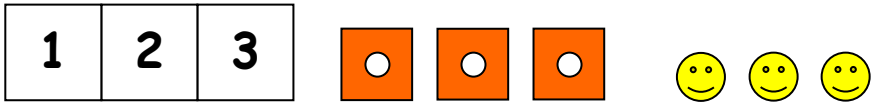
Practical Activities – Develop an interest and understanding of number

- Show an interest in number activities and counting e.g. indicates through speech or gesture a number during songs.
- Singing and games – Pupils join in with number rhymes such as Noah's Ark; the animals went in 2 by 2.
- To make groups of 2 using familiar pairs and sort according to sets e.g. socks/shoes.

Vocabulary

- Double
- Pairs
- Twos
- Ones
- Groups
- Sort

- To count in 1s:
  - Use number songs/games.
  - Use visual models to relate to numeral and objects e.g. 2, the Numicon shape and objects.



- To gain experience of doubling and become familiar with appropriate language:
  - Use a range of practical activities such as Noah's Ark, objects in sand/water, Numicon shapes or Teddy Bears Picnic.

Bear has 1 cake. Let's double it.

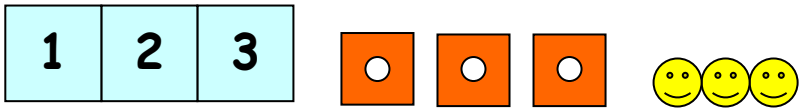
How many does he have?

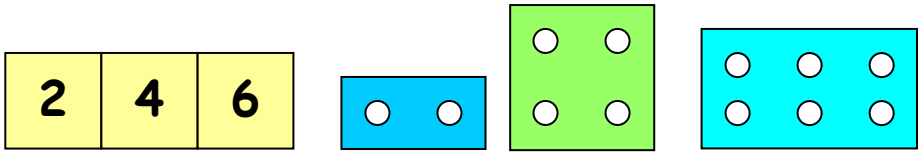
Reception (40 – 60 months)

Practical Activities – Develop understanding of multiplication

- To continue to gain experience of doubling and become familiar with appropriate language:
  - Use a range of practical activities such as Noah's Ark, objects in sand/water, Numicon shapes or Teddy Bears Picnic.

- To begin counting in 1's, 2's, 5's and 10's:
  - Use number songs/games.
  - Use visual models to relate to numeral and objects e.g. 2, the Numicon shape and objects.





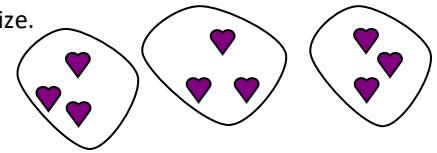
Vocabulary

- Same

Visual models to support each step:

- Numicon Shapes
- Sorting box objects
- Cubes
- Variety of toys e.g. small world
- Sorting hoops

- To make groups of objects to 10.
- To count repeated groups of the same size.



Bridging 1 and 2

Making Pairs

- To continue to make pairs or sets of familiar objects e.g.

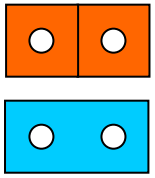
Vocabulary

- ***As Above***
- Half
- Halves
- Multiply
- Multiplied
- Hops
- Equal

Doubling

- The children learn that double means 'twice as many'. They should be given opportunities to build doubles using real objects and mathematical Equipment. Children can build numbers using pair wise patterns on 10 frames will help the children see the doubles. Mirrors are also a fun way for Children to double the quantities they build.

- Encourage the children to say the doubles as they build them.

Double 1  **is 2** 

- Provide the children with examples of doubles and non-doubles for the children to sort and explain why.

Visual models to support each step:

- Prepared number line
- Numicon number line
- Numicon Shapes
- Multi-link

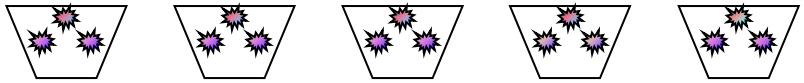
Counting in 2s and 5s

- To count on and back in 1's, 2's, 5's and 10's:
  - Counting in 2's e.g. counting socks, shoes, animal legs
  - Counting in 5's e.g. counting fingers, fingers in gloves

Use pictures/marks

- To solve multiplication problems using pictures and/or symbols:

There are 3 sweets in one bag. How many sweets are there in 5 bags?



Bridging 3 and year 1

Count in 10s

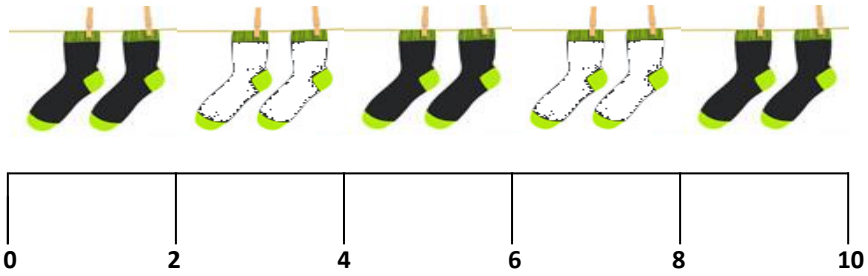
- Children count in groups of tens for the first time. They use pictures, bead strings, number lines and numicon to support their counting.
- Counting in tens on a number square will also support the children to see the similarities between the numbers when we count in tens.

Making equal groups.

- Children begin by using stories which link to pictures and concrete manipulatives to explore making equal groups and write statements such as 'there are ___groups of ___'. They will recognise and explain how they know when they are equal or not.
- Children see equal groups that are arranged differently so they understand that the groups look different but can still be equal in number.

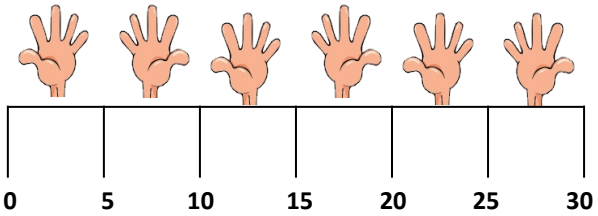
Repeated Addition (add equal groups)

- Children use equal groups to find a total. They focus on counting equal groups of 2, 5 and 10 and explore this within 50.



2 + 2 + 2 + 2 + 2 = 10
2 x 5 = 10
2 multiplied by 5
5 pairs
5 hops of 2

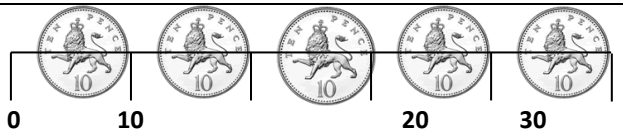
5 + 5 + 5 + 5 + 5 + 5 =30
5 x 6 = 30
5 multiplied by 6
6 groups of 5
6 hops of 5



Vocabulary

- **As Above**
- Half
- Halves
- Multiply
- Multiplied
- Hops
- Partition
- Times tables
- Division
- Tens
- Ones

10p + 10p + 10p + 10p + 10p = 50p



- Children could begin by linking this to real life, for example, animal legs, wheels, flowers in vases etc.
- Stem sentences alongside number sentences can help children link the calculation with the situation. Ensure children have the opportunity to say their sentences aloud.

Make arrays

- Children begin to make arrays by making equal groups and building them up in columns and rows. They can use a range of concrete and pictorial representations alongside sentence stems to support their understanding. Children also explore arrays built incorrectly and recognise the importance of columns and rows.

Making doubles to 20

- Children explore doubling with numbers up to 20. Reinforce understanding that 'double' is two groups of a number or an amount. Children show and explain what doubling means using concrete and pictorial representations.
- They record doubling using the sentence, 'Double ___ is ___' and use repeated addition to represent doubles in the abstract. They look at representations to decide whether that is doubling or not.

MULTIPLICATION
Year 2

Recognise equal groups

- Children describe equal groups using stem sentences to support them. It is important that children know which groups are equal and unequal, and why they are equal or unequal.
- The addition and multiplication symbols are not used yet but the use of the language for addition and multiplication will support them in understanding repeated addition and subtraction.

Make equal groups

- Children should be able to make equal groups to demonstrate their understanding of the word 'equal'.

Add equal groups

- Children begin to connect equal groups to repeated addition. (Children would have added 3 one-digit number together, therefore they can add up to 3 equal groups when each group is any one digit number). If there is more than 3 equal groups, the examples must be limited to 2s, 5s, 10s and 3s.

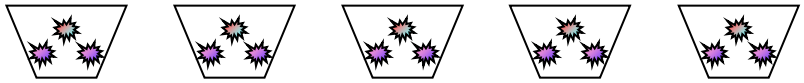
The Multiplication Symbol

- Children are introduced to the multiplication symbol for the first time. They should link repeated addition and multiplication together, using stem sentences to support their understanding.
- They should also be able to interpret mathematical stories and create their own involving multiplication.
- The use of concrete resources and pictorial representations is still vital for understanding.

Multiplication from pictures

- Children will use the multiplication symbols and work out the total from pictures.
- They should also be able to interpret a multiplication word problem by drawing images to help them solve it. You could also coins as well.

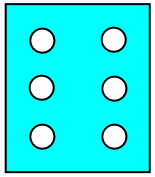
There are 3 sweets in one bag. How many sweets are there in 5 bags?



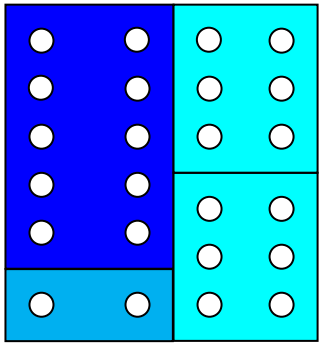
Doubling to 20

- To recall doubles to 20 and look at the corresponding halves (more able):
  - Use Numicon Shapes and multi-link as visual support.

Double 6



is 12



- To continue to count on and back in 1's, 2's, 5's and 10's:
  - Counting in 2's e.g. counting socks, shoes, animal legs
  - Counting in 5's e.g. counting fingers, fingers in gloves
  - Counting in 10's e.g. fingers, toes

X and = signs and missing numbers

- To be able to complete number sentences where a missing number is shown by a gap or symbol:

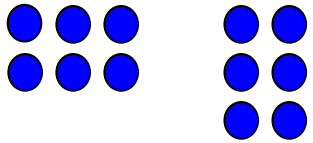
7 x 2 = □ □ = 2 x 7

7 x □ = 14 14 = 2 x □

□ X △ = 14 14 = □ x △

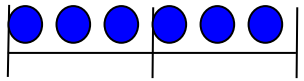
Use Arrays

- Children explore arrays to see the commutativity of multiplication facts e.g. 5 x 2 = 2 x 5
- The use of the array could be sued to help children calculate multiplication statements. The multiplication symbols and language of 'lots of' should be used interchangeably.
- To use arrays:

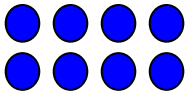


  - Proving that 2 rows of 3 has the same number of spots as 3 rows of 2.

- To use arrays on a number line:

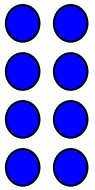


- To use arrays to solve multiplication problems (link to repeated addition):



2 x 4 = 8

4 x 2 = 8



4 x 2 = 8

2 x 4 = 8

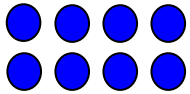
Visual models to support each step:

- Prepared number line
- Numicon number line
- Empty number line
- Numicon Shapes
- Multi-link

MULTIPLICATION

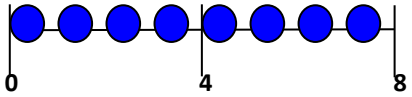
Year 2

Repeated addition



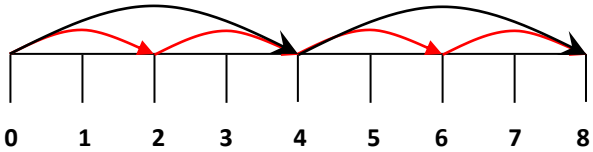
2 x 4 = 2 + 2 + 2 + 2

Link to jumps on a number line:



2 x 4 = 4 + 4

4 + 4 or 2 + 2 + 2 + 2



The 2 times table

- Children should be comfortable with the concept of multiplication so they can apply this to multiplication tables. Pictures and number tracks should be used to encourage the children to count in twos. Resources such as cubes and number pieces are important for children to explore equal groups within the 2 times table.

The 5 times table

- Children can already count in 5s from any given number. They will also have developed understanding of the 2 times table. This small step is focuses on the 5 times table and it is important to include the use of zero. Children should see the = sign at both ends of the calculation to understand that it means 'equal to'.

The 10 times table

- CHildrne have counted in 10s from any given whole number. This small step is focused on the 10 times table and it is important to include the use of zero. Children should see the = sign at both ends of the calculation to understand what it means.

Partitioning

- To solve multiplication by partitioning 'teen' numbers:

15 x 2 = (10 + 5) x 2
= 10 x 2 = 20
= 5 x 2 = 10

= 20 + 10
= 30

- To derive and recall multiplication facts for the 2, 5 and 10 times tables and the related division facts.

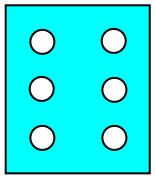
Multiplication

Year 3

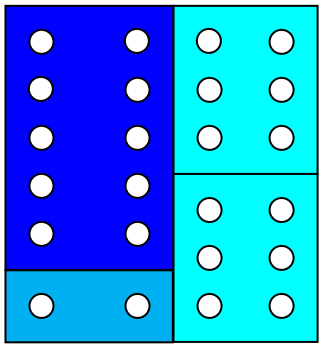
Doubling to 20 and beyond

- To continue to recall doubles to 20 and look at the corresponding halves:
  - Use Numicon Shapes and multi-link as visual support.

Double 6



is 12



Vocabulary

- **As above**
- Grid
- Times
- Multiplication
- Product
- Division
- Inverse
- Partition
- Ones
- Tens
- Addition
- Repeated

X and = signs

- To be able to complete number sentences where a missing number is shown by a gap or symbol as in Year 2 but with appropriate numbers.
- To derive and recall multiplication facts for the 2, 3, 4, 5, 6 and 10 times tables and the related division facts.
- To understand that multiplication is the inverse of division:

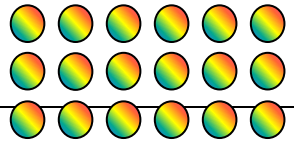
3 x 5 = 15 ⟶ **15 ÷ 5 = 3**

Equal groups

- Children recap their understanding of recognising, making and adding equal groups. This will allow them to build on prior learning and prepare them for the next small steps.

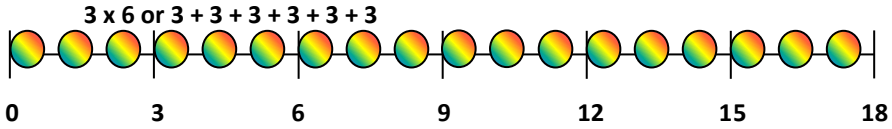
Arrays and repeated addition

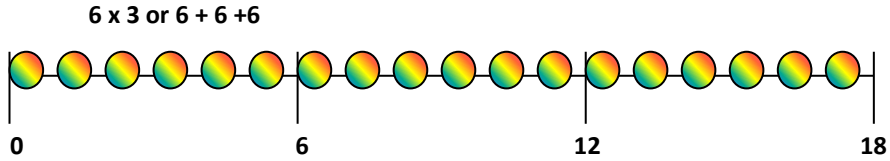
- To continue to understand multiplication as repeated addition and use arrays (as in Year 2) e.g.

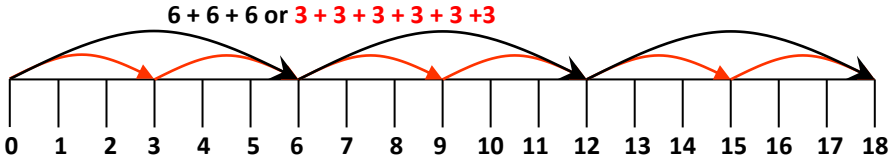


6 x 3 or 6 + 6 + 6

3 x 6 or 3 + 3 + 3 + 3 + 3 + 3







Visual models to support each step:

- Prepared number line
- Numicon number line
- Empty number line
- Numicon Shapes
- Multi-link
- Pre-drawn multiplication grids

MULTIPLICATION
Year 3

Multiply by 3

- Children draw on their knowledge of counting in threes in order to start to multiply by 3. They use their knowledge of equal groups to use concrete and pictorial methods to solve questions and problems involving multiplying by 3.

The three times table

- Children draw together their knowledge of multiplying and dividing by three in order to become more fluent in the three times table. Children apply their knowledge to different contexts.

Multiply by 4

- Building on their knowledge of the two times table, children multiply by 4.
- They link multiplying by 4 to doubling the doubling again.
- Children connect multiplying by 4 to repeated addition and counting in 4s.
- To show the multiplication of 4, children may use number pieces, cubes, containers bar models etc.

The 4 times table

- Children use knowledge of known multiplication tables (2, 3, 5 and 10 times table) and understanding of key concepts of multiplication to develop knowledge of the 4 times table.
- Children who have learnt that 3 x 4 = 12 can use understanding of commutativity to know that 4 x 3 = 12

Multiplying by 8

- Building on their knowledge of the 4 times table, children start to multiply by 8, understanding that each multiple of 8 is double its equivalent multiple of 4.
- They link multiplying by 8 to previous knowledge of equal groups and repeated addition. Children explore the concept of multiplying by 8 in different ways, when 8 is the multiplier (first number in the multiplication calculation) and where 8 is the multpicand (second number)

The 8 times table

- Children use prior knowledge of multiplication facts for 2, 3, 4 and 5 times table along with the distributive law in order to calculate unknown multiplication facts.

Comparing statements

- Children use their knowledge of multiplication and division facts to compare statements using inequality symbols. It is important that children are exposed to a variety of representations of multiplication and division, including arrays and repeated addition.

Related Calculations

- Children use known multilocation facts to solve other multiplication problems. They understand that because one of the numbers in the calculation is ten times bigger, the then answer will also be ten times bigger.
- It is important that children develop their conceptual understanding through the use of concrete manipulatives.

Pencil and Paper Procedures

Multiply 2 digits by 1 digit

- Children use their understanding of repeated addition to represent a two-digit number multiplied by a one-digit number with concrete manipulatives. They use the formal method of column multiplication alongside the concrete representation. They also apply their understanding of partitioning to represent and solve calculations. In this step, children explore

multiplication with no exchange.

- Children continue to use their understanding of repeated addition to represent a two-digit number multiplied by a one-digit number with concrete manipulatives. They more on to explore multiplication with exchange.

Scaling

- It is important that children are exposed to problems involving scaling from an early age. Children should be able to answer questions that use the vocabulary 'times as many'. Bar models are particularly useful here to help children visualise the concept. Examples and non-examples should be used to ensure depth of understanding.

How many ways

- Children list systematically the possible combinations resulting from two groups of objects. Encourage the sue of practical equipment and ensure that children take a systematic approach to each problem.
- Children should be encouraged to calculate the total number of ways without listing all the possibilities. E.g. Each t-shirt can br matched with 4 pairs of trousers so altogether 3 x 4 = 12 outfits.

MULTIPLICATION
Year 4

Multiply by 10

- Children need to be able to visualise and understand making a number ten times bigger and that 'ten times bigger' is the same as 'multiply by ten'. The language of 'ten lots of' is vital to use in this step. The understanding of the commutative law is essential because children need to see calculations such as 10x3 and 3x10 as equal.

Multiply by 100

- Children build on multiplying by 10 and see links between multiplying by 10 and multiplying by 100. Use place value coins and Base 10 to explore what is happening to the value of the digits in the calculation and encourage children to see a rule so they can begin to move away from concrete representations.

Multiply by 1 and 0

- Children explore the result of multiplying by 1, using concrete equipment. Linked to this, they look at multiplying by 0 and use concrete equipment and pictorial representations of multiplying by 0.

Multiply by 6

- Children draw on their knowledge of times tables facts in order to multiply by 6. They use their knowledge of equal groups in using concrete and pictorial methods to solve multiplication problems.

6 times table facts

- Children use known table facts to become fluent in the six times tables. For example, applying knowledge of the 3 times table by understanding that each multiple of 6 is double the equivalent multiple of 3.
- Children should also be able to apply this knowledge to multiplying by 10 and 100 (for example, knowing that 30 x 6 = 180 because they know that 3 x 6 = 18)

Multiply by 9

- Children use their previous knowledge of multiplying to become fluent in the 9 times table. They apply their knowledge in different contexts.

9 times tables facts

- Children use known table facts to become fluent in the 9 times table. For example, knowing that each multiple of 9 is one less than the equivalent multiple of 10, and use this to derive related facts.
- Children should also be able to apply the knowledge of the 9 times table when multiplying by 10 and 100.

Multiply by 7

- Children use their knowledge of multiplication to multiply by 7. They count in 7s, and use their knowledge of equal groups supported by use of concrete and pictorial methods to solve multiplication calculations and problems.
- They explore commutativity and also understand that multiplication and division are inverse operations.

7 times table

- Children apply the facts from the 7 times table (an other previously learned tables) to solve calculations with larger numbers. They need to spend some time exploring links between

multiplication tables and investigating how this can help with mental strategies for calculation e.g. 7 x 7 = 49, 5 x 7 = 35 and 2 x 7 = 14.

11 and 12 times table

- Building on their knowledge of the 1, 2 and 10 times-tables, children explore the 11 and 12 times tables through partitioning. They use Base 10 equipment to build representations of the times tables and use them to explore the inverse of multiplication and division statements. Highlight the importance of commutativity as children should already know the majority of facts from other times tables.

Multiply 3 numbers

- Children are introduced to the 'Associative Law' to multiply 3 numbers. This law focuses on the idea that it doesn't matter how we group the numbers when we multiply e.g. 4 x 5 x 2= (4x5) x 2 = 20 x 2 =40 or 4 x 5 x 2 = 4 x (5x2) = 4 x 10 = 40.
- They link this idea to commutativity and see that we can change the order of the numbers to group them more efficiently, e.g. 4 x 2 x 5= (4x2) x 5 = 8 x 5 = 40

Factor Pairs

- Children learn that a factor is a whole number that multiples by another number to make a product e.g. 3 x 5 = 15, factor x factor = product.
- They develop their understanding of factor pairs using concrete resources to work systematically, e.g. factor pairs using 12 – begin with 1 x 12, 2 x 6, 3 x 4. At this stage, children recognise that they have already used 4 in the previous calculation therefore all factor pairs have been identified.

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X and = signs

- To be able to complete number sentences where a missing number is shown by a gap or symbol as in Phase 4/5 but with appropriate numbers.
- To derive and recall multiplication facts to 10 x 10 and the related division facts.

- To identify the doubles of 2-digit numbers and use these to calculate doubles of multiples of 10 and 100 and derive the corresponding halves.

Efficient Multiplication

- Children develop their mental multiplication by exploring different ways to calculate. They partition two-digit numbers into tens and ones or into factor pairs in order to multiply one and two digit numbers. By sharing mental methods, children can learn to be more flexible and efficient.

Partitioning

- To continue to solve multiplication problems by using partitioning but with larger numbers:

44 x 6 = (40 + 4) x 6
= 40 x 6 = 240
= 4 x 6 = 24
= 240 + 24
= 264

Vocabulary

- **As above**
- Approximate
- Hundreds
- Multiple

- Continue to use visual support – number lines, Numicon Shapes etc.

Written Methods

- Children use a variety of informal written methods to multiply a two digit and a one digit number. It is important to emphasise when it would be more efficient to use a mental method to multiply and when we need to represent our thinking by showing working out.

Some example Pencil and Paper Procedures for 2 digit by 1 digit and 3 digit by 1 digit

- To use the grid method of multiplication (include making approximations) e.g. TU x U extending to HTU x U and TU x TU

23 x 7 =

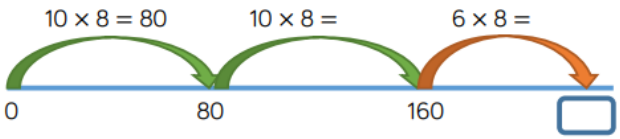
X	20	3
7	140	21

is approximately **20 x 10 = 200**

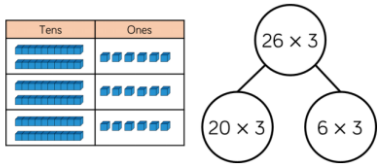
HTU x U **154 x 4**

X	100	50	4
4	400	200	16

There are 8 classes in a school.
Each class has 26 children.
How many children are there altogether?
Complete the number line to solve the problem.



Rosie uses Base 10 and a part-whole model to calculate 26 × 3
Complete Rosie's calculations.



Use Rosie's method to work out:
36 × 3
24 × 6
45 × 4

- Children also build on their understanding of formal multiplication from year 3 to move the formal short multiplication method. Children use their knowledge of exchanging tens ones for one ten in addition and apply this to multiplication, including exchanging multiple groups of tens. They use pace value counters to support their understanding.
- Children build on previous steps to represent a three digit number by a one digit number with concrete manipulatives. Teacher should be aware of misconceptions arising from 0 in the tens and ones column. Children continue to exchange groups of ten ones for tens and record this in a written method.

Correspondence Problems

- Children solve more complex problems building on their understanding from year 3 of when *n* objects relate to *m* objects. They find all solutions and notice how to use multiplication facts to solve problems.

DIVISION

0 – 3 years (16–26 months, 22-36 months) and **3 – 4 years** (30-50 months)

Practical Activities – Develop an interest and understanding of number

- Show an interest in number activities and counting e.g. indicates through speech or gesture a number during songs.
- Singing and games – Pupils join in with number rhymes, stories and songs. They take part in number games.
- To match everyday items one-one e.g. hat for each person or spoon in each bowl.

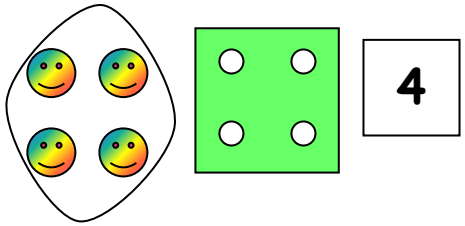
Vocabulary

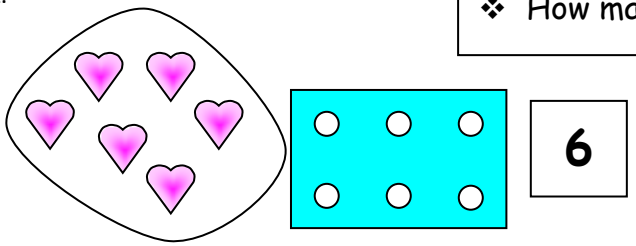
- Match
- Give
- How many?

Visual models to support each step:

- Numicon Shapes
- Variety of toys or sorting objects

- To count 'how many' objects in groups:
  - Match to Numicon Shapes and numerals as further support.





Reception (40 – 60 months)

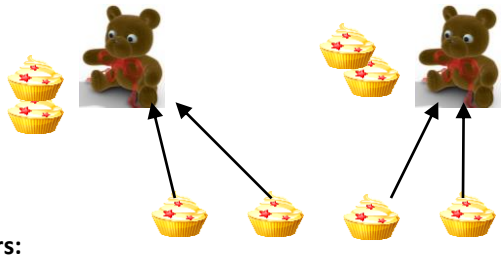
Practical and Pictorial Activities – Develop understanding of division as sharing

- To share objects e.g. raisins shared between children or 6 eggs shared between 2 nests.

Vocabulary

- Share
- Shared
- Between
- Half

- To gain experience of halving and become familiar with appropriate language:
  - Use a range of practical activities with teacher modelling such as using Numicon shapes or Teddy Bears Picnic.



Give half the cakes to each of the bears:

Visual models to support each step:

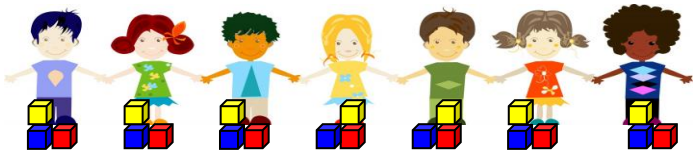
- Numicon Shapes
- Sorting box objects
- Cubes
- Variety of toys e.g. small world
- Sorting hoops

Bridging 1 and 2

Halving and Sharing

- The children will halve quantities by sharing items into 2 equal groups. The distinction between fair and unfair sharing can be used to emphasise the idea of half being as being one of 2 equal parts.
- Once children are confident at halving small quantities, they can explore sharing between 3 and 4 people. They will notice that items are left over and may come up with their onw suggestions for how to resolve this.

- To share equally e.g. sharing toys between peers.

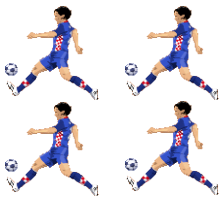


Vocabulary

- As Above

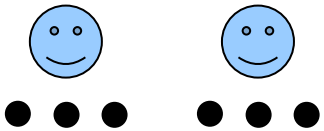
- To represent problems using pictures and symbols:

12 people get into teams of 4 to play a game of football. How many teams are there?



OR

6 sweets are shared between 2 people. How many do they have each?



Visual models to support each step:

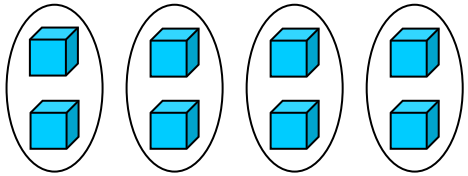
- Numicon Shapes
- Sorting box objects
- Variety of toys and objects
- Sorting hoops

Odds and Evens

The children begin to understand that quantities which can be shared into 2 equal groups with no items left are even. Those which have one left over whey they are shared into 2 equal groups are odd.

- Encourage the children to notice this structure on the number shapes and by building pair-wise patterns on the 10 frames.
- They can also explore odd and even by grouping quantities into paris. Even quantities can be grouped into pairs and odd quantities will have one left on their own when they are grouped into pairs.

DIVISION
Bridging 3 and year 1
Make equal groups (Grouping) • Children start with a given total and make groups of an equal amount. They record their understanding in sentences, not through formal division at this stage. • Children can develop their understanding of equal groups by also being exposed to numbers which do not group equally. **Sharing Equally** Children explore sharing as a model of division. They use 1:1 correspondence to share concrete objects into equal groups. Children also need to be given the opportunity to see whn a number of objects cannot be shared equally into equal groups.

DIVISION
Year 2
Make equal groups – sharing • Children divide by sharing objects into equal groups using one to one correspondence. They need to do this suing concrete manipulatives in different contexts, then move onto pictorial representations. • Children will be introduced to the ÷ symbol. They will begin to see the link between division and multiplication. **8 sweets are shared between 2 children. How many do they each have?**  **Make equal groups – Grouping** • Children divide by making equal groups. They then count on to find the total number of groups. They need to this using concrete manipulatives and pictorially in a variety of contexts. • They need to recognise the link between division, multiplication and repeated addition. **8 ÷ 2** can be modelled as:  **Divide by 2** • Children should be secure with grouping and sharing. They will use this knowledge to help them divide by 2. They will be secure with representing division as an abstract number

Vocabulary

- ***As Above***
- Division
- Divide
- Divided
- Group
- Grouping
- Sharing
- Subtraction
- Repeated

sentence using the division and equals symbols.
- Children should be able to count in 2s and know their 2 times table.

Odd and Even Numbers
- Building on from year 1, children should be able to recognise odd and even numbers.
- They will use concrete manipulatives to explore odd and even numbers and the structure of these.

Divide by 5
- During this step, children focus on efficient strategies and whether they should use grouping or sharing depending on the content of the question.
- They use their knowledge of the five times table to help them divide by 5. They will continue to see the = sign both before and after the calculation.

Divide by 10
- Children should already be able to multiply by 10 and recognise multiplies of 10. They will need to use both grouping and sharing to divde by 10 depending on the context of the problem.
- Children start to see that grouping and counting in 10s is more efficient than sharing into 10 equal groups.

÷ and = signs and missing numbers
- To be able to complete number sentences where a missing number is shown by a symbol or a gap:

6 ÷ 2 = ◯	◯ = 6 ÷ 2
6 ÷ ◯ = 3	3 = 6 ÷ ◯
◯ ÷ 2 = 3	3 = ◯ ÷ 2
◯ ÷ Δ = 3	3 = ◯ ÷ Δ

Visual models to support each step:
- Numicon Shapes
- Sorting box objects
- Variety of toys and objects
- Sorting hoops
- Multilink
- Prepared number lines

÷ and = signs and missing numbers

- To be able to complete number sentences where a missing number is shown by a symbol or a gap as in Phase 4 but with appropriate numbers.

- To understand that multiplication is the inverse of division:

3 x 5 = 15 ⟶ **15 ÷ 5 = 3**

Divide by 3

- Children explore dividing by 3 through sharing into three equal groups and grouping in threes. They use concrete and pictorial representations and use their knowledge of the inverse to check their answers.

Divide by 4

- Children explore dividing by 4 through sharing into four equal groups and grouping in fours. They use concrete and pictorial representations and their knowledge of the inverse to check their answers.

Divide by 8

- Children explore dividing by 8 through sharing into eight equal groups and grouping in eights. They use concrete and pictorial representations and their knowledge of inverse operations to check their answers.

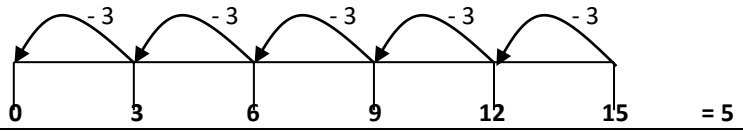
Understand Division as Sharing and Grouping

- **15 ÷ 3** can be **15 shared between 3.**
  - Use objects as in Year 2:

Vocabulary

- **As Above**
- Multiplication
- Inverse
- Array
- Remainder

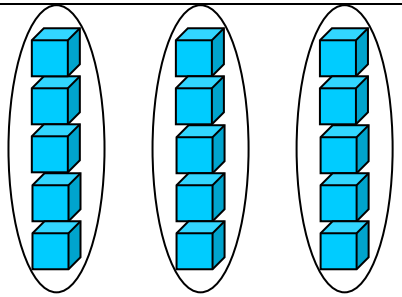
  - Use a number line:



= 5

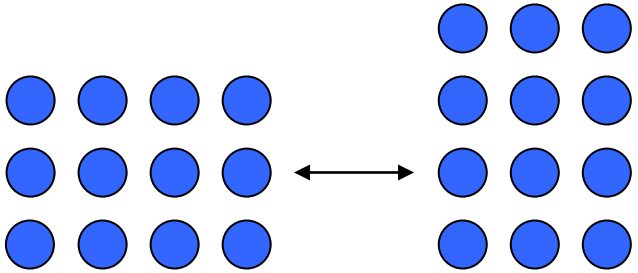
DIVISION
Year 3

- **How many 3's make 15.**
  - Use objects as in Year 2:



Arrays

- To use arrays to see that **12 ÷ 3** is the same as **12 ÷ 4**:



Divide 2 digits by 1 digit

- Children divide 2-digit numbers by a 1 -digit number by partitioning into tens and ones and sharing into equal groups. They divide numbers that do not involve exchange or remainders. It is important that children divide the tens first and then the ones.
- Children divide 2-digit numbers by a 1-digit number by partitioning into tens and ones and sharing into equal groups. They divide numbers that involve exchanging between the tens and ones. The answers do not have remainders. Children use their times tables to partition the number into multiples of the divisor.

DIVISION
Year 3
Remainders

- To use objects to solve division problems with simple remainders:

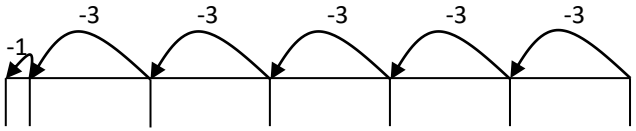
  Sharing: **16 cakes are shared between 3 children. How many cakes does each child get? How many are left over?**

Grouping: **How many groups of 3 can be made from 16 cakes? How many cakes are left over?**

Divide 2-digits by 1 digit with remainders

- Children move onto solving division problems with a remainder. Links are made between division and repeated subtraction, which builds on learning in year 2.

Sharing (repeated subtraction):



DIVISION
Year 4
Divide by 10 • Exploring questions with whole number answers only, children divide by 10. They should sue concrete manipulatives and place value charts to see the link between dividing by 1and the position of the digits before and after the calculation. Using concrete resources, children should begin to understand the relationship between multiplying and dividing by 10 as

the inverse of the other.

Divide by 100

- Children divide by 100 with whole number answers. Money and measure is a good real-life context for this, as coins can by used for the concrete stage.

Divide by 1

- Children learn what happens to a number when you divide it by 1 or itself. Using concrete and pictorial representations, children demonstrate how both the sharing and grouping structures of division can be used to divide a number by 1 or itself. Use stem sentences to encourage children to see this e.g. 5 grouped into 5s equals 1 (5 ÷ 5 = 1). 5 groped into 1s equals 5 (5 ÷ 1 = 5)

Divide by 6

- Children draw on their knowledge of times tables facts in order to divide by 6. They use their knowledge of equal groups in using concrete and pictorial methods to solve division problems.

6 times table facts

- Children use known table facts to become fluent in the six times tables. For example, applying knowledge of the 3 times table by understanding that each multiple of 6 is double the equivalent multiple of 3.
- Children should also be able to apply this knowledge to dividing by 10 and 100 (for example, knowing that 180 ÷ 60 = 3 because they know that 18 ÷6 = 3)

Divide by 9

- Children use their previous knowledge of dividing to become fluent in the 9 times table. They apply their knowledge in different contexts.

9 times tables facts

- Children use known table facts to become fluent in the 9 times table. For example, knowing that each multiple of 9 is one less than the equivalent multiple of 10, and use this to derive related facts.
- Children should also be able to apply the knowledge of the 9 times table when dividing by 10 and 100.

Multiply by 7

- Children use their knowledge of division to divide by 7. They count in 7s, and use their knowledge of equal groups supported by use of concrete and pictorial methods to solve multiplication calculations and problems.
- They explore commutativity and also understand that multiplication and division are inverse operations.

7 times table

- Children apply the facts from the 7 times table (and other previously learned tables) to solve calculations with larger numbers. They need to spend some time exploring links between multiplication tables and investigating how this can help with mental strategies for calculation e.g. 7 x 7 = 49, 5 x 7 = 35 and 2 x 7 = 14.

Divide 2 digits by 1 digit

- To be able to complete number sentences where a missing number is shown by a symbol or a gap as in Year 3 but with appropriate numbers.
- Children also build on their knowledge of dividing a 2-digit number by a 1 digit number from year 3 by sharing into equal groups. Children use examples where the tens and the ones

are divisible by the divisor e.g. 96 divided by 3 and 84 divided by 4.

- They then move on to calculations where they exchange between tens and ones.
- Children then explore dividing a 3-digit number by a 1-digit number involving remainders. They continue to use place value counters to divide in order to explore why there are remainders. Teacher should highlight, through questioning, that they remainder can never be greater than they number you are dividing by.
- Children apply their previous knowledge of dividing by a 2 digit number to divide a 3 digit number by a 1 digit number. They use place value counters and part-whole models to support their understanding. Children divide numbers with and without remainders.